PGTC 2017

TUESDAY 27 JUNE

On Quillen's conjecture (joint work with Antonio Diaz). *Dr Nadia Mazza, Lancaster University.*
Let A be the poset of nontrivial elementary abelian p-subgroups of a finite group G, ordered by inclusion. Quillen's conjecture asserts that if the associated simplicial complex $|A|$ of A is contractible, then G has a nontrivial normal p-subgroup. Quillen proved the conjecture in several cases, in particular for finite solvable groups and finite groups of Lie type in defining characteristic. Elaborating on Quillen, Aschbacher and Smith's works, A. Diaz developed a geometric method leading to an improved proof of the conjecture in the known cases. In this talk, we will review Quillen's conjecture and present ongoing joint work with A. Diaz.

Generating Graphs of Finite Groups. *Scott Harper, University of Bristol.*
The study of finite groups and their generating sets has a long and rich history. The generating graph is a recently introduced combinatorial gadget which neatly encodes many interesting generation properties that are the focus of current research in this area. More precisely, the vertices of the generating graph of a finite group G are its non-identity elements and two vertices are adjacent if the elements generate G. Many natural graph theoretic invariants of this graph have an interesting group theoretic interpretation. For example, the generating graph of G has no isolated vertices if and only if every non-identity element of G is contained in a generating pair. In 2008, Breuer, Guralnick and Kantor conjectured that the generating graph of G has no isolated vertices if and only if every proper quotient of G is cyclic. In this talk we will discuss existing work on the generating graph and report on some recent progress towards the conjecture of Breuer, Guralnick and Kantor.

Generation of finite simple groups. *Carlisle King, Imperial College London.*
Given a finite simple group G, it is natural to ask how many elements are needed to generate G. It has been shown that all finite simple groups are generated by a pair elements. A natural refinement is then to ask whether the orders of the generating elements may be restricted: given a pair of integers (a, b), does there exist a pair of elements (x, y) generating G with x of order a and y of order b? If such a pair exists, we say G is (a, b)-generated. I will explore some past results regarding $(2, 3)$-generation as well as a new result on $(2, p)$-generation for some prime p.

The McKay conjecture for S_{2^n}. *Jasdeep Kochhar, Royal Holloway University of London.*
Let G be a finite group, and let $P \in \mathrm{Syl}_2(G)$ with normaliser $N_G(P)$ in G. Let $\mathrm{Irr}_{2'}(G)$ denote the set of odd-degree ordinary irreducible characters of G. In 1972, McKay conjectured that $|\mathrm{Irr}_{2'}(G)| = |\mathrm{Irr}_{2'}(N_G(P))|$. This conjecture was proved by Malle and Späth in 2015.

In 2016, Giannelli proved that the restriction map $\mathrm{Res}^G_{N_G(P)} : \mathrm{Irr}_{2'}(G) \to \mathrm{Irr}_{2'}(N_G(P))$ is a bijection, when G is the symmetric group S_{2^n}. In this talk I will give an explicit description of this map at the level of modules. I will provide further examples of groups such that the restriction map gives the required bijection.

Conjugation Probabilities for Finite Groups. *William O'Donovan, Royal Holloway, University of London.*
Given a finite group G, one can ask what the probability that two elements of G (chosen independently and uniformly at random) are conjugate is. In my talk, I will give a general overview of this problem, providing motivation and illustrating some of the big results in the area. I will then present results based

on my recent work analysing the conjugation probability for specific classes of groups: iterated wreath products (including Sylow subgroups of symmetric groups) and generalised symmetric groups.

Wednesday 28 June

Constructing Majorana representations. *Madeleine Whybrow, Imperial College London.*
Majorana theory is an axiomatic framework in which to study objects related to the Monster group and its $196,884$ dimensional representation, the Griess algebra. The objects at the centre of the theory are known as Majorana algebras and can be studied either in their own right, or as Majorana representations of certain groups.

I will discuss my work developing an algorithm in GAP to construct the Majorana representations of a given group. This work is based on a paper of . Seress and is joint with M. Pfeiffer.

Albert Algebras and the Exceptional Groups of Lie Type. *Yegor Stepanov, Queen Mary University of London.*
The 27-dimensional exceptional Jordan algebra (or Albert algebra) of 3×3 Hermitian matrices over the octonions turns out to be a fruitful object for some exceptional groups of Lie type. It was shown that the group of the automorphisms of the Jordan algebra over any field is isomorphic to the Chevalley group F_4 over the same field. The stabilizer of the determinant, which is represented by a certain cubic form, is actually a group of type E_6, and its twisted version is obtained by considering those elements of E_6 over the quadratic field, which preserve certain Hermitian form. In this talk we are going to look at the constructions of these (and some other) finite groups defined by the automorphisms of the Albert algebra.

Broué's conjecture, modular representation theory and perverse equivalences: an overview. *Stefano Sannella , University of Birmingham.*
The study of representation theory of a finite group G over a field F of positive characteristic carries many questions that have not been answered yet. Most of them can be defined as global/local conjectures: in various formulations, they state that the representation theory of G is strongly controlled by its p-local subgroups. This short talk aims to give a general overview of the matter, its recent development and will focus on one of those questions in particular: Broué's Abelian Defect Group Conjecture.

Realizing saturated fusion systems. *Athar Warraich, University of Birmingham.*
Given a finite group G and a finite p-subgroup T, a fusion category $\mathcal{F}_T(G)$ is a category whose objects are subgroups of T and whose morphisms are conjugation maps induced by elements in G. A saturated fusion system $\mathcal{F}$, over a finite p-group T, is a category whose objects are subgroups of T and whose morphisms are injective group homomorphisms satisfying certain properties. Every saturated fusion system is a fusion category for some T and finite G. It is exotic if there is no finite G such that $T \in \mathrm{Syl}_p(G)$. In this talk we discuss how to construct G for certain exotic fusion systems.

Fusion over a Sylow p-subgroup of $GL_4(p)$. *Raul Moragues Moncho, University of Birmingham.*
Fusion in a finite group refers to conjugacy between subgroups of a group, such as the action of a group on its Sylow p-subgroups. A modern way of studying this is by starting with a p-group and building a saturated fusion system over it. The fusion is controlled by some subgroups, called essential, which are self-centralizing subgroups of S whose automorphism group has a nice structure. We will discuss these concepts, and apply them to a Sylow p-subgroup of $GL_4(p)$ and how this relates to $GU_4(p)$.

Certain irreducible characters over a normal subgroup. *Noelia Rizo Carrión, University of Valencia.*
The celebrated Howlett-Isaacs theorem [1] on groups of central type solved a conjecture proposed by Iwahori and Matsumoto in 1964: if Z is a normal subgroup of a finite group G, $\lambda \in \mathrm{Irr}(Z)$ is a G-invariant complex irreducible character of Z, and the induced character λ^G is a multiple of a single $\chi \in \mathrm{Irr}(G)$, then G/Z is solvable. This theorem, proved in 1982, is one of the first applications of the Classification of Finite Simple Groups to Representation Theory.

In this talk we present a generalization of this important theorem taking into account actions by automorphisms and we study certain induced characters whose constituents have all the same degree.

References

[1] R.B. Howlett, I.M. Isaacs, On groups of central type, Math. Z. **179** (1982) 555-569.
[2] G. Navarro, N. Rizo, Certain irreducible characters over a normal subgroup, Journal of Group Theory, ISSN (Online) 1435-4446, ISSN (Print) 1433-5883, DOI: https://doi.org/10.1515/jgth-2016-0052.

Relational complexity for finite permutation groups. *Bianca Loda, University of South Wales.*
Relational complexity is an invariant of a finite permutation group and was introduced by Cherlin in 1996. It can be rather difficult to compute the relational complexity of any given permutation group. In this talk we will give a definition of relational complexity, we will consider some interesting examples, and we will give some ideas of how to calculate this invariant for some important classes of groups. We will focus particularly on primitive groups.

Restriction of irreducible representations to maximal subgroups. *Nathan Scheinmann, EPFL.*
Let G be a simply connected simple algebraic group over an algebraically closed field K. If K has characteristic 0, the irreducible representations of G are well understood. If K has characteristic $p > 0$, most of the representation theory of G is unknown. In the latter case, a way of studying the simple KG-modules is by restricting them to a maximal subgroup H of G. I will present the case where G is of type E_6 and H a maximal subgroup of G of type F_4 given by the fixed points of the (Dynkin) graph automorphism of G.

Invariant forms on irreducible modules of simple algebraic groups. *Mikko Korhonen, EPFL.*
Let G be a simple linear algebraic group over an algebraically closed field K of characteristic $p \geq 0$ and let V be an irreducible rational G-module with highest weight λ. When V is self-dual, a basic question to ask whether the image of the representation $\rho : G \to \mathrm{GL}(V)$ is contained in a symplectic group on V or an orthogonal group on V. In other words, when does V have a non-degenerate G-invariant alternating bilinear form or a non-degenerate G-invariant quadratic form?

If $p \neq 2$, the answer is well known and easily described in terms of λ. In the case where $p = 2$, we know that if V is self-dual, it always has a non-degenerate G-invariant alternating bilinear form. However, determining when V has a non-degenerate G-invariant quadratic form is a classical problem that still remains open. In this talk, I will give an introduction to this problem, and present some recent results which settle the problem for certain λ. For example, an answer can be given in the case where G is of classical type and λ is a fundamental highest weight.

The essential dimension of algebraic groups using lattice theoretic techniques. *Gareth Case, Lancaster University.*
The essential dimension of an algebraic object is roughly speaking, the number of algebraically independent parameters needed to "describe" that object. It was first introduced by Buhler and Recihstein in 1997 for Galois field extensions, but since has been studied for algebraic groups. Using data given by integral representations and lattices, one can attain bounds on the essential dimension of a particular type of algebraic group, namely extensions of finite groups by algebraic tori.

Thursday 29 June

Tilting modules for SL_2. *Samuel Martin, University of York.*
Recently I have been able to determine exactly when a module from a certain class of rational SL_2 modules is a tilting module, over an algebraically closed field of positive characteristic. In this talk I will briefly introduce these modules, give the result, which has a nice combinatoric formulation in terms of the p-adic expansion of the highest weights, and an idea of how to prove it.

Geometric properties relating to Beilinson-Bernstein localisation. *Richard Mathers, University of Oxford.*
In recent years, Ardakov and Wadsley have been interested in extending the classical theory of Beilinson-Bernstein localisation to different contexts. The classical proof relies on fundamental geometric properties of the dual nilcone of a semisimple Lie algebra; in particular, finding a nice desingularisation of the nilcone and demonstrating that it is normal. I will attempt to explain the relationship between these properties and the proof, and discuss some areas of my own work, which focuses on proving analogues of these results in the case where the characteristic of the ground field K is bad.

Affine Kac-Moody groups and presentations. *Karina Kirkina, University of Warwick.*
Affine Kac-Moody groups over finite fields are in a sense the "smallest" infinite analogues of finite groups of Lie type. They are finitely generated but their standard presentation is infinite. I will define these groups, and then go on to explain why they in fact have finite presentations of bounded size, and show how to construct small presentations for them by piecing together presentations of the finite groups.

Rearrangement Groups of Connected Spaces. *Nayab Khalid, University of St Andrews.*
We define groups of homeomorphisms of self-similar topological spaces - 'rearrangement groups'. We study which properties these groups inherit from the topological space they act on. In particular, we discuss how we can find a generating set for such groups that stems from the basic open sets of the topological space.

The Null Space of a Cayley Graph over a Cyclic Group. *Ali Al-Tarimshawy, University of East Anglia.*
In this work we will introduce some properties of Cayley graphs for cyclic groups. We are interested in the Null Space of the adjacency map of such Cayley graphs. We will give some results about the Null Space of such Cayley graphs in term of character sums.

Base sizes of quasisimple groups and Pyber's conjecture. *Melissa Lee, Imperial College London.*
A *base* of a permutation group G acting on a set Ω is a subset of Ω whose pointwise stabiliser in G is trivial. The minimal base size of G is denoted by $b(G)$. A well-known conjecture made by Pyber in 1993 states that there is an absolute constant c such that if G acts primitively on Ω, then $b(G) < c \log |G| / \log n$, where $|\Omega| = n$. After over 20 years and contributions by a variety of authors, the conjecture was established in 2016 by Duyan, Halasi and Maróti. In this talk, I will cover some of the history of Pyber's conjecture, especially in the context of primitive linear groups, and present some results on the determination of the constant c for bases of quasisimple linear groups.

Width questions for finite simple groups. *Alex Malcom, Imperial College London.*
Width questions concern covering a given group G with elements from a normal subset, using products of minimal length. For example can we write every group element as a product of two or perhaps three elements from a fixed conjugacy class? Such problems often require the application of interesting results from character theory. In particular, we will consider finite simple groups and classes of elements of prime order.

Bivariate zeta functions of groups associated to unipotent group schemes. *Paula M Lins de Araujo, Universitaet Bielefeld.*
Zeta functions are important tools in asymptotic group theory. In this talk I will give a brief introduction to zeta functions associated to groups and then I will talk about bivariate representation and conjugacy class zeta functions of nilpotent groups which are associated to unipotent group schemes.

The structure of induced simple modules for 0-Hecke algebras. *Imen Belmokhtar, Queen Mary, University of London.*
In this talk we shall be concerned with the 0-Hecke algebra; its irreducible representations were classified and shown to be one-dimensional by Norton in 1979. The structure of a finite-dimensional module can be fully described by computing its submodule lattice. We will discuss how this can be encoded in a generally much smaller poset given certain conditions and state new branching rules in types B and D.

Friday 30 June

Finite groups containing locally maximal product-free sets of a given size. *Chimere Anabanti, Birkbeck, University of London.*
In 2006, Giudici and Hart asked the question: which finite groups contain locally maximal product-free sets of a given size $k \geq 1$? They classified all finite groups that contain locally maximal product-free sets of sizes 1 and 2, and some of size 3. Their conjecture was that if a finite group G contains a locally maximal product-free set of size 3, then $|G| \leq 24$. In this talk, we prove this conjecture. Moreover, we give a discussion on the general case argument.

Locally quasi-convex convergence groups. *Pranav Sharma, Lovely Professional University.*
The notion of convergence groups originated from the theory of the abelian groups equipped with some infinite sums. We report on the problem of the characterisation of the class of reflexive locally compact convergence groups. We define convergence group to be locally quasi convex if each filter which converge to zero contains a quasi convex set. We prove that if a convergence group is c-reflexive then it must be locally quasi convex and hence we obtain that in contrast to the topological case locally compact abelian convergence groups do not lie in the class of locally quasi-convex convergence groups. Finally, we present the role of the summability theory in the extensions of the Pontryagin duality theory beyond the realm of locally compact abelian groups.

Degree of commutativity of infinite groups. *Motiejus Valiunas, University of Southampton.*
For a finite group F, its degree of commutativity is defined to be the probability that two elements of F chosen at random commute. This concept has been recently generalised to finitely generated infinite groups G, and it seems to be closely related to growth of G. In particular, it has been conjectured that all groups of exponential growth have degree of commutativity zero. In the talk I will outline the construction and proofs of why degree of commutativity has to be zero for two particular classes of groups of exponential growth. Methods used in the proofs give further insight into combinatorial properties of these groups.

Character degrees of finite groups. *Prof. Dr. Gunter Malle, TU Kaiserslautern.*
The degrees of the irreducible complex representations of a finite group constitute an important and much studied invariant. They are known to satisfy various arithmetic properties, but surprisingly many basic questions concerning them are still open. We will give an introduction to several interesting open questions, put them into a wider perspective and present recent results.